# THEORY OF 4-PORT NONRECIPROCAL CIRCUIT
## --- FILTER AND CIRCULATOR ---

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## Abstract

The general formula for 4-port nonreciprocal circuit is given. The effects of circuit structures and physical properties of resonator on the characteristics of the circuit are investigated. The limitations of available frequency range are clarified.

## Introduction

So far a large number of studies concerned with magnetically tunable ferrimagnetic resonance filters have been reported. Only a few of them are studies on nonreciprocal filters in the sense that there is a very small attenuation in one direction of propagation, and a large attenuation in the reverse direction. However, in the case of stripline, the available frequency range of nonreciprocal filters is narrow: about 1.3:1[1,2].

In the 1972 IEEE G-MTT, we proporsed a new nonreciprocal 4-port circuit shown in Fig.1[3]. The circuit operates over a multi-octave frequency range as a nonreciprocal bandstop filter, as a nonreciprocal bandpass filter and as a magnetically-tunable resonance type circulator. An example of the experimental results of this 4-port circulator is shown in Fig.2a, where the center frequency insertion loss in the passband and that in the stopband are defined in Fig.2b.

The complete analysis of the 4-port nonreciprocal circuit has not been reported yet. The purpose of this report is to give the theoretical formula for the response of 4-port nonreciprocal circuit and to investigate the effects of the circuit structure and the physical properties of the resonator on the characteristics of the circuit, in particular at the resonant frequency.

## General Formula

The circuit now in consideration consists of two directional couplers and an X-form orthogonal coupling with a magnetic resonator placed in the center of the circularly polarized RF magnetic field.

The insertion loss in the passband, that in the stopband and the net dissipation loss of the resonator can be obtained through the analysis of the X-form orthogonal coupling. Here the detail of the derivation will be omitted and only the results are presented as follows:

$$IL_{BPF} = \left| 1 + \frac{1}{Z_+} \right|^2 \qquad (1)$$

$$IL_{BSF} = \left| 1 + Z_+ \right|^2 \qquad (2)$$

$$L = \frac{\mathcal{R}e\, 2Z_+}{\left| 1 + Z_+ \right|^2} \qquad (3)$$

where $Z_{\pm} = j\frac{1}{2Z_0}\left( \omega L_c + \frac{X}{\mp 1 + \frac{\omega_t}{\omega} + j\alpha} \right)$,

$Z_0$ = characteristic impedance of stripline, $L_c$ = self inductance of coupling line, $\omega_t$ = radian frequency connected with H, demagnetization factor and $\omega_m$, $\omega_m$ = radian frequency connected with the saturation magnetization $M_s$, $\alpha$ = Landau-lifshitz phenomenologycal loss term of resonator, $X = \omega_m A$, A = inductance depending on the volume and position of the resonator and the geometrical configuration of an orthogonal coupling.

## Characteristics

We will investigate the characteristics of the circuit, in particular, at the resonance frequency. At resonance, we get

$$IL^r_{BPF} = \left| 1 + \frac{1}{Z^r_+} \right|^2 \simeq ( 1 + \alpha Q_f )^2 \qquad (4)$$

$$IL^r_{BSF} = \left| 1 + Z^r_+ \right|^2 \simeq \left( 1 + \frac{1}{\alpha Q_f} \right)^2 \qquad (5)$$

$$L^r = \frac{\mathcal{R}e\, 2Z^r_+}{\left| 1 + Z^r_+ \right|^2} \simeq \frac{2\alpha Q_f}{( 1 + \alpha Q_f )^2} \qquad (6)$$

and the reverse loss of the bandpass filter and that of the bandstop filter:

$$RL^r_{BPF} = \left| 1 + \frac{1}{Z^r_-} \right|^2 \simeq 1 + \left( \frac{2Q_f}{2k_r + 1} \right)^2 \qquad (7)$$

$$RL^r_{BSF} = \left| 1 + Z^r_- \right|^2 \simeq 1 + \left( \frac{2k_r + 1}{2Q_f} \right)^2 \qquad (8)$$

where the index r means resonance, $Q_f = 2Z_0/X$, $k_r = \omega_r L_c/X$ and the approximate expressions hold in the case where $k_r^2/Q_u Q_f \ll 1$ ($Q_u = 1/2\alpha$). The characteristics of nonreciprocal bandpass filter and the net dissipation loss of the magnetic resonator at resonance are shown in Fig.3. Also shown in Fig.4 the center frequency insertion losses and the center frequency reverse losses, in the case where $k_r$ increases from 10 to 80.

The dissipation loss of the resonator goes through the maximum 1/2 at a value of $Q_f = 1/\alpha$, after which it decreases and at last approaches a value of zero with increasing $Q_f$. In relatively low regions of $\alpha Q_f$, even if an $\alpha$ of resonator has a smaller value, the center frequency reverse loss in the passband decreases and that in the stopband increases with increasing $k_r$. These effects are considered to degrade the characteristics of the 4-port circuit in higher regions of available frequency range. The center frequency insertion loss in the passband increases with increasing $\alpha$, while the center frequency insertion loss in the stopband decreases in spite of increasing the dissipation loss. Thus, there exists an optimum value of external Q.

The circuit structure is fixed, the center frequency reverse loss in the passband is not always increased and that in the stopband is not always reduced, though the resonator with smaller $\alpha$ is used. Thus, it is a key point whether $Q_f$ and k depending on $M_s$ increase or decrease. Consequently, for 4-port circulator of this type, the magnetic materials which have a large value of the product of $Q_u$ and $M_s$ are of much importance as a magnetic resonator. Also the circuit structure which reduces a value of the ratio $L_c/A$ is required. In such a case as the ratio $L_c/A$ can be reduced sufficiently, it is considered that good performances of the circuit may be obtained by using even a polycrystalline magnetic resonator, if the product $Q_u \cdot M_s$ is adequately chosen.

## Limitations

It is of great importance in the theory and the practice to clarify the limitations of available range and 3-dB bandwidth. The requirements are here introduced as follows:

$$IL^r_{BPF},\quad RL^r_{BSF} \lesseqq n^2,$$
$$IL^r_{BSF},\quad RL^r_{BPF} \geqq N^2 \tag{9}$$

where $2 \geqq n \geqq 1$, $N^2 \gg 1$. Under the conditions restricted by (9) and in the case where $n \geqq 1 + 1/(N-1)$, the lower and upper limitation of the fractional 3-dB bandwidth are approximately given by the following inequality:

$$\frac{N}{Q_u} \lesssim \frac{\Delta f}{f_r} \lesssim \frac{2}{Nk_r} + \frac{1}{Q_u} \tag{10}$$

where $\Delta f$ is the bandwidth between 3-dB attenuation points.

In the case of the ellipsoidally shaped resonator, it is a well-known the fact that the lower limitation $f_L$ of magnetically tunable range is given by $f_L > N_t f_m$, where $f_m = \omega_m/2\pi$, $N_t$ = transverse component of magnetizing factor. The coupling impedance X is considered to be independent of frequency over an extremely wide range. The directional coupler with coupled-transmission-lines has a very wide range, if it is adequately designed[4]. Therefore we get the upper limitation $f_H$ of the available frequency range after due consideration to (4) - (9):

$$f_H \lesssim 2\frac{A}{L_c} f_m Q^*_u \cdot \mathrm{Min}\{\, n-1,\ \frac{1}{N-1}\,\} \times \mathrm{Min}\{\, \sqrt{n^2-1},\ \frac{1}{\sqrt{N^2-1}}\,\} \tag{11}$$

where $Q^*_u$ = unloaded Q of resonator at frequency $f_H$. The lower limitation $f_L$ can be lowered by using a disk-shaped resonator and a low-$M_s$ resonator. As is obvious from (11), the upper limitation $f_H$ can be extended by using a resonator which has a large value of $Q_u M_s$ and by designing a coupling structure which has a large ratio of A to $L_c$. Example of calculations is shown in Fig. 4.

## Conclusion

The general formula for 4-port nonreciprocal circuit is obtained. The results of complete analysis show that even a polycrystalline resonator could be useful for the nonreciprocal circuit of this type, if the product of $Q_u$ and $M_s$ has a sufficiently large value. Moreover the limitations of the fractional 3-dB bandwidth and the available frequency range are made clear.

## References


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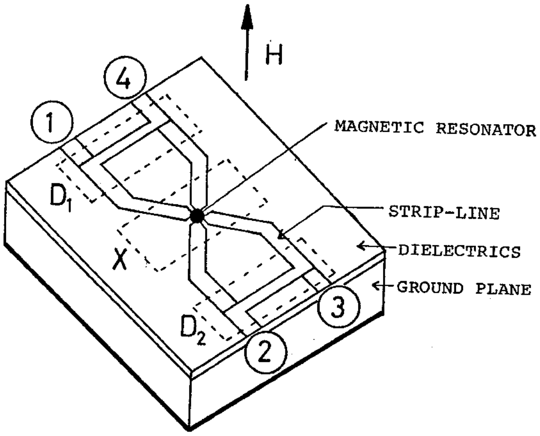


Fig. 1 Schematic 4-port nonreciprocal circuit
$D_1$, $D_2$: 3-dB directional coupler
X: X-form orthogonal coupling
H: external dc magnetic field.

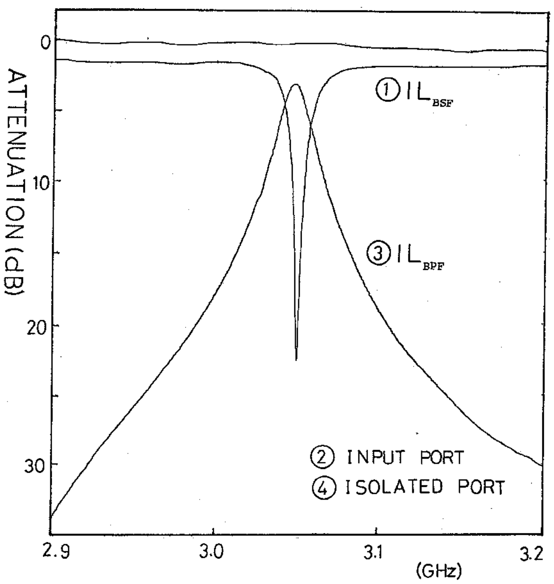


Fig. 2a Output response of the 4-port circulator. A dc magnetic field is tuned to the YIG sphere resonance and fixed.

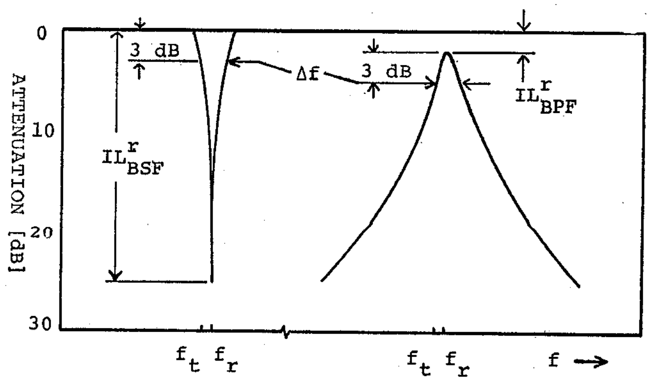


Fig. 2b Definition of filter response parameters

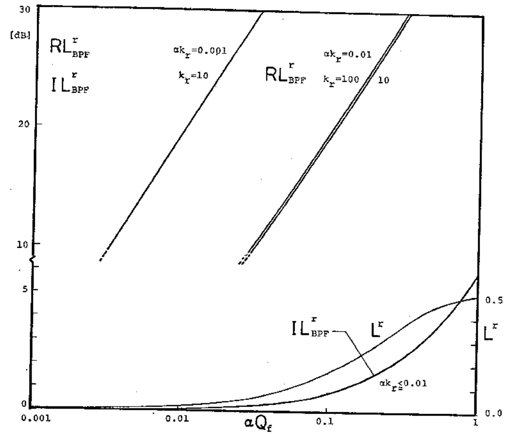


Fig. 3 Insertion loss in the passband and dissipation loss of 4-port nonreciprocal circuit as a function of $\alpha Q_f$ at resonance.

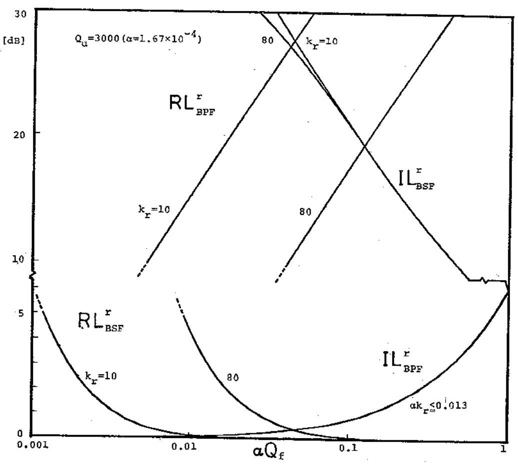


Fig. 4 Insertion losses and reverse losses of 4-port nonreciprocal circuit in the case where $k_r$ increases from 10 to 80.

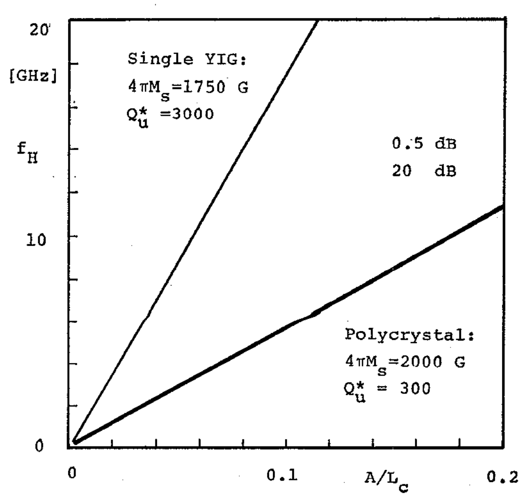


Fig. 5 Upper limitation of available range. $f_H$ for $IL^r_{BPF}$, $RL^r_{BSF} \leqq 0.5$ dB and for $IL^r_{BSF}$, $RL^r_{BPF} \geqq 20$ dB.